

#1) The differential equation model of a nonlinear time-invariant system is: $\ddot{y} = -(\dot{y}^2 + 4)y + 5u$.

- a) Given $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$, write the **state equation** of the system, as $\begin{cases} \dot{x} = f(x, u) \\ y = x_1 \end{cases}$.
- b) Draw a **Simulink model** of the system.
- c) Find the **equilibrium/operating point** of the system, for $y = 0$ fixed, (Hint: set all derivatives equal to zero).
- d) Find the **linearized state model** of the system at the equilibrium/operating point: $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $u = 0$.
- e) Find the **transfer function** $G(s) = \left. \frac{Y(s)}{U(s)} \right|_{IC=0}$ of the linearized system.
- f) Find the **poles and zeros** of the above transfer function $G(s)$, and check the **stability** of the system.
- g) **Explain** what the following Matlab commands do?
 - I. `[xe,ue,ye,dxe]=trim('sim_model',[],[],0,[],[],1)` (Compare to: `[X,U,Y,DX]=TRIM('SYS',X0,U0,Y0,IX,IU,IY)`)
 - II. `[A,B,C,D]=linmod('sim_model',xe,ue)` (Compare to: `[A,B,C,D]=LINMOD('SYS',X,U)`)
- h) **Write** a short Matlab code to:
 - I. express the transfer function $G(s)$, found in part (f),
 - II. find the poles “p”, zeros “z”, and DC-gain “G0” of $G(s)$,
 - III. find the equivalent linear state model “sys” of $G(s)$,
 - IV. Check the controllability and observability of the above linear state model “sys”.