Engineering Models I
Homework Assignment #3

Instructions:

1. In addition to this HW3 document, you will also need the two excel files posted with HW3: ProjectileData.xlsx and HydrogenPeroxide.xlsx. Put both of this files in your current MATLAB directory.
2. **Show your work! Make sure you include MATLAB commands.**
3. It is fine to work with other students, but what you turn in must be your own work - not something copied from someone else.

<u>Problem 1</u>: Curve Fitting – Trajectory of Projectile

The excel file, ProjectileData, has three columns of data: time, distance, and height. Import each of these columns into MATLAB using either xlsread or the Import Data tool. The variable distance represents measurements of the x-position (horizontal position) of the projectile over time and the variable height represents measurements of the y-position (vertical position) of the projectile over time.

The equations for the x and y position of a projectile launched at an angle of θ (rad or degrees) with an initial velocity of V_0 (m/s) are:

$$x_{position} = [V_0 \cos(\theta)]t$$

$$y_{position} = -\frac{1}{2} g t^2 + [V_o \sin(\theta)]t + y_0 \qquad \text{Note: } g = 9.81 \text{ m/s}^2$$

(a) Plot **time** on the x-axis and **distance** on the y-axis. Add axis labels (with units) and a title to your plot. We know that the x-position of the projectile increases linearly with time. So use the curve fitting tool to fit a 1st order polynomial (line) to the distance data. Display the equation for the fitted polynomial on your graph with 5 significant digits. Then copy and paste your plot in the space below.

MATLAB PLOT:

(b) Plot **time** on the x-axis and **height** on the y-axis. Add axis labels (with units) and a title to your plot. We know the height of the projectile follows a parabolic (2nd order) curve. So use the curve fitting tool to fit a 2nd order polynomial (quadratic) to the height data. Display the equation for the fitted polynomial on your graph with 5 significant digits. Then copy and paste your plot in the space below.

MATLAB PLOT:

(c) Look at the numerical coefficient for the squared term in the fitted polynomial for the **height** data. Theoretically, this coefficient should be equal to -1/2*g. How close is it? Calculate a percent error using the following formula with -1/2*g as actual value:

$$\%Error = \frac{Estimated - Actual}{Actual}$$

Show Calculations:

% Error = ______________________

(d) The numerical coefficient for the linear term in the fitted polynomial for **height** should be approximately $V_0 \sin(\theta)$ and the numerical coefficient for the linear term in the fitted polynomial for **distance** should be approximately $V_0 \cos(\theta)$. Enter the values below then solve for the initial velocity, V_0, and the launch angle, θ.

$V_0 \sin(\theta)$ = ______________________

$V_0 \cos(\theta)$ = ______________________

θ = ________________________ (include units)

V_0 = ________________________ (include units)

Show Calculations:

<u>Problem 2</u>: Curve Fitting – Decomposition of Hydrogen Peroxide – Estimating Reaction Rates from Experimental Data

First order chemical reactions can be modeled using exponential functions. Hydrogen Peroxide (H_2O_2) decomposes as a 1st order reaction into water and oxygen gas:

$$2H_2O_2 \rightarrow 2H_2O + O_2$$

The concentration of hydrogen peroxide decreases exponentially according to the following equation:

$$C(t) = C_0 e^{-kt}$$

C(t) = Concentration at time t (M or mols/L)
C_0 = Initial Concentration (M)
k = Reaction rate (s^{-1})

In lab, we investigated the decomposition of hydrogen peroxide in air which was a very slow reaction. It decomposes much, much faster in the presence of a catalyst such as Iodide (I^-).

The HydrogenPeroxide.xlsx file has the results of five experiments measuring the concentration of hydrogen peroxide using an Iodide catalyst at 5 different temperatures. We will use curve fitting to estimate the reaction rate, k, for each of these five temperatures.
If we take the natural log of the concentration equation, we have:

$$\ln\big(C(t)\big) = -kt + \ln(C_0)$$

- For each of the five temperatures, plot **time** on the x-axis and **ln(C(t))** on the y-axis. Create five separate plots. **Remember, in MATLAB ln(C) is log(C)!**
- Next use the curve fitting tool to fit a line to the data. Display the curve fit equation with five significant digits. The slope of the line should be roughly equal to –k as long as the measurements are good. Enter your estimated reaction rate values in the table below.
- Include your plots in the space indicated (make sure they are labeled appropriately) and the MATLAB commands used to generate one of the five plots.

Absolute Temperature (K)	Estimated Reaction Rate, k (s^{-1})
280	
285	
290	
295	
300	

MATLAB COMMANDS TO GENERATE ONE OF THE PLOTS:

FIVE MATLAB PLOTS SHOWING CURVE FIT:

<u>Problem 3</u>: Curve Fitting – Decomposition of Hydrogen Peroxide – Estimating Activation Energy from Experimental Data

The reaction rate, k, depends on the temperature according to Arrhenius' Equation (also exponential):

$$k = Ae^{-E_a/(RT)}$$

A = Frequency Factor (s^{-1})
E_a = Activation Energy (J/mol)
R = Ideal Gas Constant = 8.314 (J/(mol*K))
T = Absolute Temperature (K)

We will again apply the very nice trick of taking the natural log of this equation which gives:

$$\ln(k) = \frac{-E_a}{R} \cdot \left(\frac{1}{T}\right) + \ln(A)$$

Using your five values from the previous problem, plot 1/T on the x-axis and ln(k) on the y-axis. Again, do a linear curve fit. Using the fact that the slope of your line should be approximately

–Ea/R, determine a value for the activation energy, Ea, for the decomposition of hydrogen peroxide in the presence of an Iodide catalyst.

MATLAB COMMANDS:

MATLAB PLOT:

Ea = ______________________ (include units)