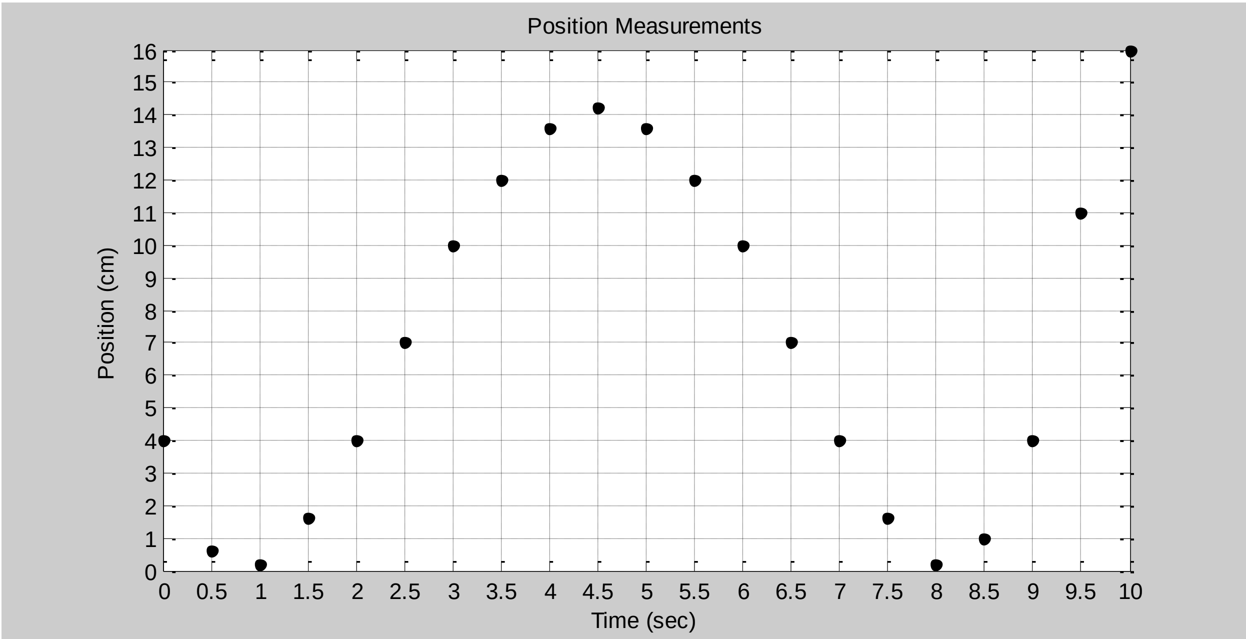
Engineering Models II: HW#4

<u>Problem 1</u>: Use the ***diff*** command in MATLAB to find the <u>first</u> and <u>second</u> derivative of the following function with respect to t. Include both your results and your MATLAB commands.

$$d = e^{-2t}\cos(5t)$$

<u>Problem 2</u>: The graph below shows position measurements (in cm) collected every 0.5 seconds over a 10 second interval of time.



(a) Using a $\Delta t = 0.5$ sec, estimate the velocity at t = 3 sec using the 2-point estimate and the 3-point estimate for derivative. **Be sure to show your work and include units!**

2-point Estimate of Velocity at t = 3 sec: __________
3-point Estimate of Velocity at t = 3 sec: __________

(b) Using the estimate for 2nd derivative and a $\Delta t = 0.5$ sec, estimate the acceleration at t = 3 sec. Again, show work!

Estimate of Acceleration at t = 3 sec: ______

(c) What could be changed to improve the accuracy of the derivative estimates?

<u>Problem 3</u>: In Models I, we graphed the equation for damped harmonic motion. If the response is ***underdamped***, the displacement of the mass can be modeled mathematically as follows:

$$d(t) = d_0 e^{-\alpha t} \cos(\omega t)$$

$$\alpha = \frac{B}{2M}$$

$$\omega = \sqrt{\frac{K}{M} - \frac{B^2}{4M^2}}$$

$$B < 2\sqrt{K \cdot M}$$

d: displacement of the mass in meters (m)

d_0: initial displacement of the mass in meters (m)

ω: the frequency of oscillation (rad/s)

M: mass (kg)

K: spring constant (N/m)

B: damping coefficient (N·s/m)

(a) Write a function in MATLAB that:

- Has one input argument, dt, and no output arguments
- Creates variables K, B, and M, and sets them to the following values:
  ```
  K = 200; B = 4; M = 0.5;
  ```
- Calculates α and ω.
- Calculates a range of t-values starting at 0, incrementing by dt, and ending at 5/α
- Creates a vector of displacement measurements at the t-values (assume $d_0 = 1$).
- Uses the displacement measurements to estimate the velocity using the 3 PT estimate for derivative
- Uses the displacement measurements to estimate acceleration
- Creates another set of t-values with a much finer increment than dt
- Calculates displacement, actual velocity, and actual acceleration over this second range of t-values.
- Uses subplot to plot displacement in one sub-window, velocity and the velocity estimate in a second sub-window, and acceleration and acceleration estimate in a third sub-window.
- Includes titles and labels (with appropriate units) for each sub-window.
- **Submit your code with your HW!**

(b) Run your function for dt = 0.1 and paste the resulting figure below.

(c) Run your function for dt = 0.025 and paste the resulting figure below.

(d) Compare the two graphs in terms of accuracy of the estimates. What do you notice about the accuracy of the estimates around t = 0 and the accuracy of the estimates closer to the end time? Why does this occur?

(e) Why is 5/α a good end time for calculating and plotting the displacement?

Problem 4: Sensor noise can cause serious problems when doing numerical differentiation or numerical integration using the measured date from the sensor. Download the HW4.mat file from Blackboard and save it in your current MATLAB directory. HW4.mat has a vector of noisy displacement measurements (disp_noise) taken at increments of 0.025 seconds.
You are basically going to repeat Problem 3 using the noisy displacement data.

(a) Write a MATLAB function that:

- Has no input and output arguments
- Creates variables K, B, and M, and sets them to the following values:
  `K = 200; B = 4; M = 0.5;`
- Calculates α and ω.
- Calculates a range of t-values starting at 0, incrementing by 0.025, and ending at 5/α
- Includes the command: load HW4 to load in the vector of noisy measurements called disp_noise.
- Uses the ***noisy*** displacement measurements to estimate the velocity using the 3 PT estimate for derivative
- Uses the **noisy** displacement measurements to estimate acceleration
- Creates another set of t-values with a much finer increment than dt
- Calculates displacement, actual velocity, and actual acceleration over this second range of t-values.
- Uses subplot to plot actual displacement ***and the noisy measurements*** in one sub-window, velocity and the velocity estimate in a second sub-window, and acceleration and acceleration estimate in a third sub-window.
- Includes titles and labels (with appropriate units) for each sub-window.
- **Submit your code with your HW!**

(b) Run your function and paste the resulting figure below.

(c) Comment on the results. How noisy does the displacement data appear to be? What is the effect of the noise on the estimates for velocity and acceleration?