Engineering Models II: Homework #5

For each of these problems include both your MATLAB commands and results. Be sure to include units in your answers!

<u>Problem 1</u>: The equations of motion for the x and y position of a projectile fired at an initial velocity of Vo (m/s) and at an angle of θ (degrees) assuming an initial position of (0,0) are:

$$y = -\frac{1}{2} \cdot gt^2 + V_0 \sin(\theta)\, t$$

$$x = V_0 \cos(\theta)\, t$$

$$g = 9.81\ m/s^2$$

(a) Take the 1st derivative of y with respect to time, t, and set it equal to zero to find the time, t, at which the height of the projectile is at a maximum. Use the 2nd derivative test to verify that the height is indeed maximized for your time, t.
(b) Using your results from part (a), find the time at which the height is maximized and the maximum height assuming an initial velocity of Vo = 70 m/s, a launch angle of 30°, and g = 9.81 m/s^2.
(c) Verify your results graphically by plotting the projectile height vs. time (using the values given in part (b)) and marking the maximum height using the Data Cursor Tool. Make sure your plot is labeled and includes units.

<u>Problem 2</u>: In HW3, you determined that the time at which the projectile hits the ground, assuming an initial height of zero, is t = 2Vosin(θ)/g.

(a) Plug the impact time into the equation for the x-position of the projectile to get the range of the projectile (how far it travels horizontally). Take the 1st derivative of x to find the angle, θ, which maximizes the range. Use the 2nd derivative test to verify that the range is indeed maximized for your angle, θ. ***Note: leave Vo, g, and θ as variables for this part.***
(b) Using your results from part (a), find maximum horizontal range assuming an initial velocity of 70 m/s and g = 9.81 m/s^2.
(c) Verify your results graphically by plotting the range vs. launch angle (using the values given in part (b)) and marking the maximum range using the Data Cursor Tool. Launch angle should be on the x-axis and range from 0 to 90°. Make sure your plot is labeled and includes units.

<u>Problem 3</u>: The linear velocity of a piston connected to a crank of radius, r, with a rod of length, L, is given by:

$$\mathrm{v(t)} = -r\omega \sin(\theta) - \frac{r^2 \omega \sin(2\theta)}{2L}$$

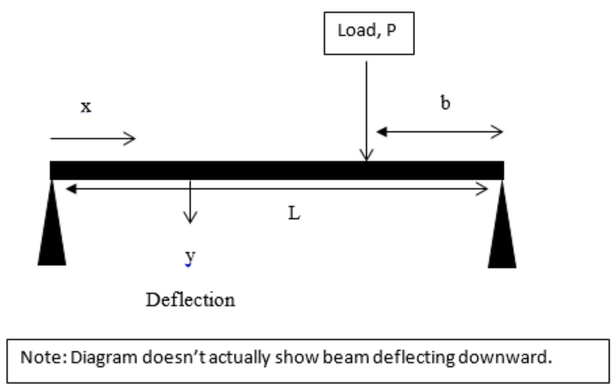
ω is the angular velocity of the crank (radians/sec) and θ is the crank angle. Suppose r = 0.025 m, L = 0.4 m, and ω = 60 r.p.m.

(a) Convert the angular velocity, ω, to radians per second.
(b) Plug the values for r, L, and ω into the linear velocity equation. Using the 1st derivative, find the crank angle, θ, at which the linear velocity of the piston, v(t), is at a maximum and the crank angle at which the linear velocity of the piston, v(t), is at a minimum. Use the 2nd derivative test to prove which crank angle results in a maximum linear velocity and which crank angle results in a minimum linear velocity.
(c) For the angles derived in part (b), find the maximum and minimum linear velocity of the piston.
(d) Verify your results graphically by plotting linear velocity vs. crank angle. The crank angle, θ, should go on the x-axis and vary from 0 to 2π radians. Mark the maximum and minimum linear velocities using the data cursor tool. Make sure your plot labeled and includes units.

<u>Problem 4</u>: The deflection in meters, y, of a beam that is simply supported on both ends and supporting a load, P (Newtons, N), is given by:

$$y = \frac{Pbx}{6LEI}(L^2 - x^2 - b^2) \qquad\qquad 0 < x < (L - b)$$

L is the length of the beam (m), E is the modulus of elasticity (N/m^2), I is the moment of inertia (m^4), b is the distance of the load from the right side of the beam (m), and x is the horizontal distance from the left side of the beam (m).



(a) Suppose L = 5 m, b = 1 m, E = 5×10^{10} N/m^2, I = 8×10^{-5} m^4, and P = 120 N. Plug these values into the equation for deflection, then find the value for x that maximizes the deflection. Prove that your value for x does indeed maximize deflection using the 2nd derivative test.
(b) Using your results from part (a), find the maximum deflection in meters.
(c) Verify your answers in part (b) graphically by plotting the deflection y (on the y-axis) vs. x as x varies from 0 to (L–b) meters. Mark the peak deflection using the Data Cursor Tool. Make sure your plot includes labels with appropriate units.